A novel statistical model for predicting matrix cracking in high temperature polymer composite laminates

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This paper studied the progressive matrix cracking in high temperature polymer composite laminates that could be used for next generation high speed transport airframe structures and aircraft engine components exposed to elevated temperatures. Damage mechanisms of matrix cracking were identified by X-ray radiography at room temperature and in-test photography technique at high temperature. It was found that the non-deterministic scenario is always involved in the procedure of transverse matrix cracking. Monte Carlo simulations using experimentally obtained materials properties were applied to simulate the multiple transverse cracking and compared with the experiment data. Finally, a novel statistical model combining Weibull theory with shear lag model was proposed to predict the matrix cracking based upon the previously obtained probability density function of crack spacing. It is shown that the predictions of this statistical model agree well with the experimental results and can be used to have an in-depth understanding of the random matrix cracking problem in composite laminates.

1. Introduction

In recent years, high temperature polymer composites are receiving increasing attention due to the fact that they are good candidates for applications to the critical parts of aircrafts such as transport airframe structures and aircraft engine components undergoing high temperature environments. Therefore, the damage and failure mechanisms of these high temperature composites need to be well understood before they can be confidently used in industry. Experimental observations have shown that for multidirectional laminates the typical modes of damage usually involve matrix cracking in the off-axis plies, delamination at the interface and fiber breakage [1–5]. Since the initiation and development of transverse matrix cracking in 90° layers is the first stage of damage under static loading and can degrade the durability of composite laminates by inducing other more severe damage modes, extensive experimental and theoretical works can be found in the literature [6–10].

Meanwhile, extensive research has also been conducted on characterization of fragmentation of coatings or the single-fiber fragmentation test [11–16]. Wheeler and Osaki [11] studied the exclusion zone and discussed the existence of different cracking regimes (random cracking, midpoint cracking and delamination) of single-fiber fragmentation test. Wagner and Eitan [12] determined the effective interfacial shear stress in the single-fiber fragmentation test. Hui *et al.* [13] showed that the theory of Curtin [14] only provides an excellent approximation for moderate and large values of the Weibull shape parameter and they derived an accurate closed form solution for the evolution of fiber fragments in a single fiber composite. Handge [15, 16] also considered the nonlinear elastic stress transfer effects and the existence of two scaling regimes for two-phase composite systems.

Transverse matrix cracking happens when the applied load is increased above a threshold value that is determined by the laminate lay-ups, the strength and residual stress in the transverse layers. It is found that for brittle materials cracks initiate at the free edges of the specimens and propagate across the thickness and width of the transverse layers instantaneously [8, 9]. When the crack density is low, the transverse cracks are distributed almost randomly in the length of the specimens, while scattering of the crack spacing becomes smaller at higher stresses. Basically this random process is always involved in the transverse cracking of composite laminates. In contrast, the deterministic models predict that the new cracks should happen in the middle of each crack segment where the maximum stress is applied [1, 6, 7]. It is widely accepted that this discrepancy between experimental observation and

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theoretical prediction comes from the inherent material defects such as microcracks, voids and interface degradation between fibers and matrix. Therefore it is necessary to introduce non-deterministic models to investigate this random multiple cracking problem [17–20].

Mainly two approaches have been introduced to study this non-deterministic transverse cracking problem. Some models are based on the assumption of the probability density function (PDF) of material flaws [19, 20] while others based on the assumption of the PDF of the strength in transverse layers [8, 9]. In general, it is difficult to introduce an accurate PDF of material flaws representing the actual material defects studied unless microscale or even nanoscale features of materials are captured appropriately. Alternatively, it is relatively easier to start from the PDF of transverse strength because it can be obtained directly from experimental data.

Laws and Dvorak [10] proposed a progressive damage model based upon the statistical fracture mechanics for transverse cracking in cross-ply composites. The shear lag model was used in their analysis and gave good prediction for stiffness reduction. In order to analyze progressive matrix cracking, they assumed three cases for the creation of a new transverse crack between two consecutive cracks, which has different failure probability functions. Even though their theoretical prediction showed a good agreement with the experimental results from Wang [21], this model still could not be generally used for the lack of physical understanding of multiple random crackings.

Okabe and Takeda [22] studied the mechanisms of multiple matrix cracks and proposed a model using the variations in matrix-rich region in the cross section as flaws initiating matrix cracking. They assumed the statistical distribution of matrix cracking stress and compared the theoretical predictions with experimental results at only room temperature.

Berthelot and Le Corre [8] presented a statistical analysis of transverse cracking and delamination under static loading. They used a generalized stress model and a pseudo-normal distribution of strength in the 90° layer to predict the progression of transverse cracking. The parameters of this pseudo-normal distribution were obtained by curve fitting with the experimental results from Highsmith and Reifsnider [2]. Obviously, this distribution function is empirical and could not be extended to other materials.

Since the PDF of transverse strength plays an important role in controlling the multiplication of the transverse cracking, the choice of the distribution function has to be not only mathematically easier but also in agreement with experimental data directly.

This paper investigated the transverse matrix cracking of a typical high temperature composite material systematically at various temperatures. Experimental data at both room and high temperatures show that the transverse strengths follow Weibull distribution, which verified the previous discussion. Monte Carlo simulation technique using the experimentally obtained parameters was applied to simulate this non-deterministic multiple cracking problem and good agreement with experimental data was achieved. Finally, a novel statistical model combining Weibull theory with shear lag model was proposed to predict the relation between applied stress and crack density in transverse layer. It is shown that this statistical model based upon the previously obtained PDF of crack spacing can be used to address the random cracking process with minimum assumptions.

2. Progressive damage model of cross-ply laminates

Daniel and Lee [6] proposed a closed form solution which gives the transverse crack density, stress distributions and reduced stiffness of transverse layers as well as the entire laminate as a function of the applied load and lamina properties. By using shear lag assumption and theory of elasticity, they analyzed the cross-ply laminate as shown in Fig. 1.

The axial stresses in 0° and 90° layers are σ_{x1} and σ_{x2} respectively, which are given by

$$\sigma_{x1} = \frac{E_1}{E_x}\left[1 + \frac{E_2 h_2 \cosh(\alpha\ell/2 - \alpha x)}{E_1 h_1 \cosh(\alpha\ell/2)}\right]\bar{\sigma}_x + \left[1 - \frac{\cosh(\alpha\ell/2 - \alpha x)}{\cosh(\alpha\ell/2)}\right]\sigma_{r1} \quad (1)$$

$$\sigma_{x2} = \left[1 - \frac{\cosh(\alpha\ell/2 - \alpha x)}{\cosh(\alpha\ell/2)}\right]\left(\frac{E_2}{E_x}\bar{\sigma}_x + \sigma_{r2}\right) \quad (2)$$

where E_1 is the Young's modulus along fiber direction, E_2 is the Young's modulus along transverse direction, σ_{r1} is the residual stress at 0° layer, σ_{r2} is the residual stress at 90° layer, ℓ is the crack spacing of the segment, $E_x = \frac{h_1 E_1 + h_2 E_2}{h_1 + h_2}$ and α is shear lag parameter,

$$\alpha^2 = \frac{(h_1 + h_2)E_x}{h_1 h_2 E_1 E_2}H, \quad H = \frac{3G_{12}G_{23}}{h_1 G_{23} + h_2 G_{12}}$$

where G_{12} is the in-plane shear modulus and G_{23} is the out-of-plane shear modulus of the unidirectional lamina. Basically, Equation 2 shows the distribution of axial stress in the transverse layer. If the stress σ_{x2} reaches the transverse strength of the laminate F_{2t} at location x, a new crack will be created in the transverse layer. Thus the laminate stress $\bar{\sigma}_x$ required to produce

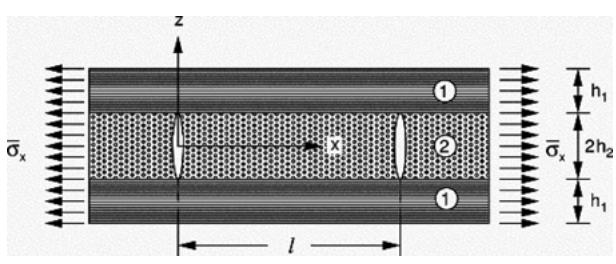

Figure 1 Element of cross-ply laminate with transverse cracks in 90° layer under uniaxial loading.

a crack spacing ℓ_0 can be obtained by

$$F_{2t} = \left(1 - \frac{1}{\cosh\frac{\alpha\ell_0}{2}}\right)\left(\frac{E_2}{E_x}\bar{\sigma}_x + \sigma_{r2}\right) \qquad (3)$$

3. Experimental
3.1. Material
The material in this investigation was IM7/977-3 carbon/epoxy, a high temperature polymer composite, and supplied in prepreg form. Laminated panels were fabricated using autoclave press. Two unidirectional laminates were made. $[90_8]$ is for tests of transverse properties, and $[0_4]$ for tests of longitudinal and shear properties. Two cross-ply panels were made: $[0/90_4]_s$ and $[0/90_2]_s$ for studying of progressive matrix cracking. All the panels were inspected by ultrasonic C-scan and found to have no discernible defects. The fiber volume ratio is 0.65, which was determined by using image analysis of photomicrographs of transverse (to the fibers) cross sections of the composite. The $[90_8]$ panels were cut with diamond saw into 229 mm × 25.4 mm coupons for transverse tensile tests. $[0_4]$ panels were cut into 229 mm × 12.7 mm coupons for longitudinal tensile tests and 229 mm × 12.7 mm coupons with the fibers oriented at 10° with the loading axis for shear tests.

To prevent fiber breakage during gripping, end tabs were used. For room temperature test, the glass/epoxy was used as tabs. It was bonded to the laminate with 907 blue Epoxy-patch kit manufactured by the Dexter Corporation. Before test, the 907 Epoxy-patch was cured at room temperature for 24 h. For high temperature test, the aluminum sheet was used as tabs, and it was bonded to the laminate with FM300 adhesive films (Cytecfiberite Inc.) cured at 149°C.

All tests were performed on an INSTRON 1331 hydraulic test machine with an INSTRON 8500 controller. The environmental chamber attached to the INSTRON was used for high temperature tests.

The properties of unidirectional laminate were fully characterized at room temperature 24°C, 93°C and 149°C respectively. The properties are tabulated in Table I.

TABLE I Mechanical properties of unidirectional laminate

	24°C	93°C	149°C
Longitudinal modulus E_1 (GPa)	191	186	179
Transverse modulus E_2 (GPa)	9.89	9.48	8.93
In-plane shear modulus G_{12} (GPa)	7.79	7.45	6.53
Major Poisson's ratio ν_{12}	0.35	0.35	0.34
Minor Poisson's ratio ν_{21}	0.018	0.018	0.017
Longitudinal tensile strength F_{1t} (MPa)	3250	3167	2961
Ultimate longitudinal tensile strain ε_{1t}^u	0.0156	0.015	0.0139
Transverse tensile strength F_{2t} (MPa)	61.62	54.52	50.59
Ultimate transverse tensile strain ε_{2t}^u	0.006	0.0058	0.0056
In-plane shear strength F_6 (MPa)	75.2	61.4	49.63
Ultimate in-plane shear strain γ_6^u	0.014	0.0132	0.0121

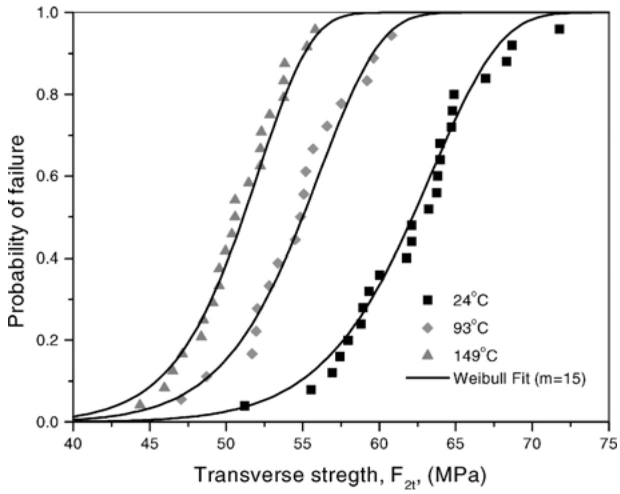

Figure 2 Cumulative transverse strength distribution at different temperatures.

3.2. Statistical distribution of transverse strength
There are two factors that determine transverse cracking in the transverse layer. One is applied stress in the transverse layer, and the other is the transverse strength of the lamina. As discussed before, transverse strength distribution due to random material defects needs to be investigated first to study the non-deterministic progressive damage of transverse layers.

In our work, a $[90_8]$laminate was used to measure transverse strength distribution parameters. Both room temperature and high temperature uniaxial tensile tests were run. Every specimen was carefully aligned before the test to avoid unexpected bending failure. The experimental data of transverse strength distribution at three temperatures are shown in Fig. 2.

It was found that Weibull distribution agrees very well with the experimental data of transverse strength distribution and the probability of failure of a uniformly loaded specimen predicted by Weibull distribution is:

$$P(F_{2t}) = 1 - \exp\left[-\left(\frac{F_{2t} - F_{2t}^u}{\beta}\right)^m\right] \qquad (4)$$

where m is the shape parameter, β is the scale parameter and F_{2t}^u is the stress threshold. m and β can be determined by experiments.

From our experimental data, we found that the change of shape parameter m of transverse strength distribution is insensitive to temperatures, which is equal to 15, and only mean transverse strengths change with temperatures.

3.3. Progressive cracking of cross-ply laminates
Damage development was studied in two cross-ply lay-ups: $[0/90_4]_s$ and $[0/90_2]_s$. For every lay-up, experimental data of applied stress versus crack density were obtained both at room temperature 24°C and 149°C. For room temperature test, transverse cracking was monitored by X-radiography enhanced with a penetrant

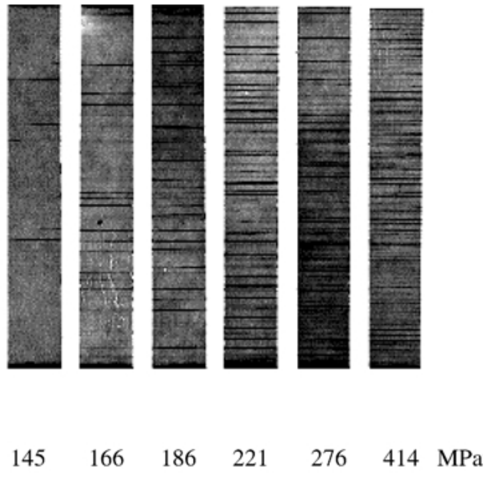

Figure 3 X-radiographs of IM7/977-3 $[0/90_4]_s$ laminate under uniaxial tensile loading at various applied stress levels.

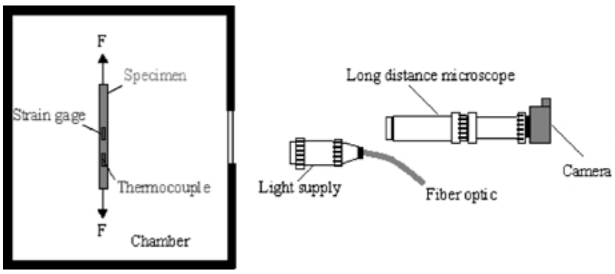

Figure 4 Schematic of high temperature test apparatus.

opaque to X-rays. At every load level, the specimen was taken off the machine and exposed to soft X-rays to obtain radiographs. Fig. 3 shows the typical progression of cracking in the 90° layer of $[0/90_4]_s$ laminate by the X-radiography at various applied stress levels. However, for high temperature tests, neither X-radiography nor replica technique [16] could work, because both of them need to take the specimen out of the chamber and would change the thermal environment of the high temperature test. Instead, a high temperature in-test photography set up as shown in Fig. 4 was developed to inspect the damage in the transverse layer. The edge of the specimen was carefully polished before the test. A long distance microscope (Infinity Co.) was used to monitor the crack multiplication from the edge during the test. A Nikon camera that was connected to the microscope was used to record the crack distribution along the gage length. Two typical crack images at the edge are shown in Fig. 5. These crack images at both room and high temperatures show that the crack spacings are not uniform, especially at low stress levels. In other words, the new cracks did not initiate in the middle of each segments as the deterministic theory predicts. We will find in next section that this is due to the statistical effects of transverse strength.

It is also worthwhile to mention that our experimental observation essentially agrees with the power law theory at different regimes of fragmentation of composites discussed by Mezin [23], Wagner [12] and Handge [15, 16], because at low stress levels, new cracks appear randomly. On the other hand, when the stress is high, new cracks will form near to the fragments' centers. Thus, these two different cracking mechanisms account for the power law theory with different exponents. Experimental data of applied stress vs. crack density for $[0/90_4]_s$ and $[0/90_2]_s$ are shown in Figs 6 and 7 respectively.

4. Monte Carlo simulation of progressive transverse cracking

4.1. Conceptual assessment of statistical transverse cracking

Equation 2 gives the axial stress distribution between two cracks in transverse layer. When the applied stress is not large, which means the crack spacing is much bigger than the thickness of transverse layer, the axial stress in the central region of the segment is very flat. However, experimental results show that transverse strength distribution is not uniform but follows the Weibull distribution. In the segment shown in Fig. 8, the transverse strength at each point falls inside a Weibull dominated band. With the increase of the applied load, the point whose stress first reaches its corresponding strength in the band will be the location of next crack. Since the transverse strength varies from point to point, the new crack could be anywhere except the two end region, which is dependent on the stress and strength distribution. It means that if statistical effects are taken into consideration for transverse cracking and the crack spacing is still large compared to the thickness of the transverse layer, the new crack unnecessarily happens in the middle point of each segment, and comparatively minor statistical variation in transverse strength distribution will lead to statistical variation of crack spacing. Consequently, transverse strength distribution is crucial to further investigating the transverse cracking problem and should not be chosen without experimental verification.

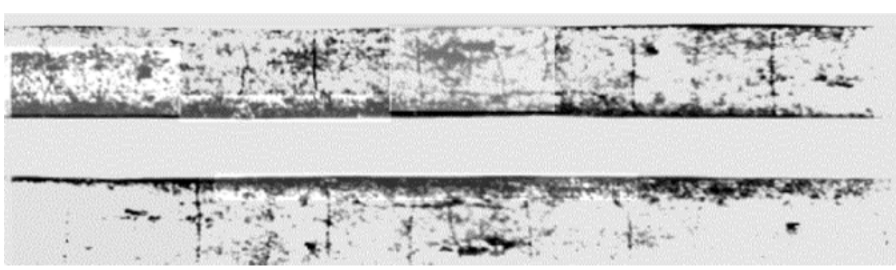

Figure 5 Typical crack image from the edge at 149°C.

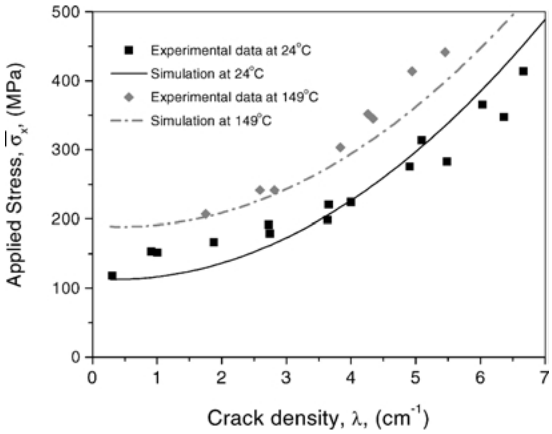

Figure 6 Simulations and experimental data of applied laminate stress versus crack density for $[0/90_4]_s$ at 24°C and 149°C.

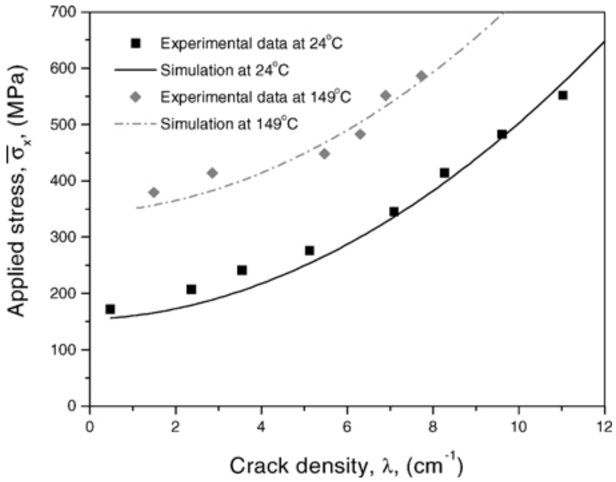

Figure 7 Simulations and experimental data of applied laminate stress versus crack density for $[0/90_2]_s$ at 24°C and 149°C.

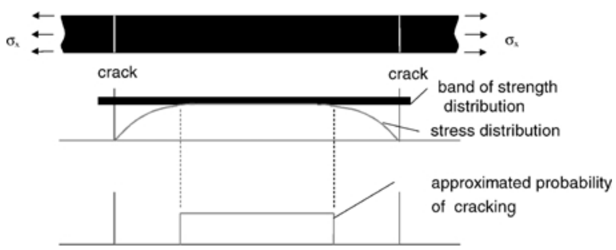

Figure 8 Schematic of stress distribution and strength distribution band between two existing cracks at low applied stress.

4.2. Monte Carlo simulation of transverse cracking

Since matrix cracks initiate from the material defects such as microcracks, voids and the interface between fibers and matrix, which are randomly distributed in the composite materials, it is necessary to introduce statistical approaches such as Monte Carlo simulation technique to deal with this non-deterministic damage process.

Monte Carlo simulations performed in this study were based upon the statistical distribution of transverse strength obtained experimentally, and differed from those of others, such as Wang [21] and Calard [20]. In other words, instead of assuming flaw size distributions, the crack multiplication can be obtained by partitioning the initial gage length into small equal elements and assigning strength randomly to each element in accordance with the Weibull distribution from experiments without any flaw size assumptions. In total, 5,000 elements were used in the simulations and the Weibull shape parameter used was 15 from our experiments. When the applied stress is low, simulations show that the location of new cracking is not necessary in the middle of the segments, but scattered in the central region discussed in Section 4.1. Simulations also show that midpoint cracking becomes increasingly dominant with the increase of the applied stress. This confirms our previous discussion. Simulation results for $[0/90_4]_s$ and $[0/90_2]_s$ at room and 149°C are shown in Figs 6 and 7 respectively, which agree very well with experimental data.

5. Theoretical model

Since the relation between applied stress and crack density is crucial to characterize the damage behavior inside the transverse layer, many models have been proposed. Nevertheless, few of them correlated their predictions to the statistical distributions of transverse cracking systematically. In this section, we take advantage of Weil and Daniel's work [24] on fracture probabilities in nonuniformly stressed materials and developed a statistical model that explores the intrinsic relations between the applied stress in the laminate and the PDF of crack spacing.

5.1. Prediction of single segment

From Weibull distribution theory, the probability of failure of one volume under a nonuniform stress σ is [24]

$$P(\sigma) = \begin{cases} 1 - \exp\left[-\int_V \left(\dfrac{\sigma - \sigma_u}{\sigma_0}\right)^m \dfrac{dV}{V_0}\right] & \\ \quad = 1 - \exp(-B), & \sigma \geq \sigma_u \\ \quad 0, & \sigma < \sigma_u \end{cases} \tag{5}$$

where $B = \int_V (\frac{\sigma - \sigma_u}{\sigma_0})^m \frac{dV}{V_0}$ is risk of rupture, σ_0 is the scale parameter, σ_u is the stress threshold, m is the shape parameter, and V_0 is the reference volume.

This theory can be applied to the typical segment in transverse layer of composite laminates shown in Fig. 1. The probability of failure of the segment ℓ under uniaxial loading is

$$P(\bar{\sigma}_{2x}) = \begin{cases} 1 - \exp\left[-\int_\ell \left(\dfrac{\sigma_{2x} - \sigma_u}{\sigma_0}\right)^m \dfrac{d\ell}{\ell_0}\right] & \\ \quad = 1 - \exp(-B), & \sigma \geq \sigma_u \\ \quad 0, & \sigma < \sigma_u \end{cases} \tag{6}$$

where $\bar{\sigma}_{2x} = \frac{E_2}{E_x}\bar{\sigma}_x + \sigma_{r2}$ is the applied stress in the transverse layer, ℓ_0 is the reference length and σ_{2x} is the local axial stress at location x, which is given by

$$\sigma_{2x} = \left[1 - \frac{\cosh(\alpha\ell/2 - \alpha x)}{\cos(\alpha\ell/2)}\right]\bar{\sigma}_{2x} \tag{7}$$

Substituting Equation 7 into Equation 6, and assuming $\sigma_u = 0$, we have

$$\begin{aligned} B &= \int_0^\ell \left(\frac{\bar{\sigma}_{2x}}{\sigma_0}\right)^m \left[1 - \frac{\cosh(\alpha\ell/2 - \alpha x)}{\cosh(\alpha\ell/2)}\right]^m \mathrm{d}x/\ell_0 \\ &= \frac{\ell}{\ell_0}\left(\frac{\bar{\sigma}_{2x}}{\sigma_0}\right)^m I(\alpha\ell, m) \end{aligned} \tag{8}$$

where

$$I(\alpha\ell, m) = \int_0^1 \left[1 - \frac{\cosh(\alpha\ell/2 - \alpha\ell t)}{\cosh(\alpha\ell/2)}\right]^m \mathrm{d}t \tag{9}$$

The mean applied stress $(\bar{\sigma}_{2x})_m$ of this segment is obtained from the probability theory:

$$\begin{aligned} (\bar{\sigma}_{2x})_m &= \int_0^\infty \frac{\mathrm{d}P(\bar{\sigma}_{2x})}{\mathrm{d}\bar{\sigma}_{2x}} \bar{\sigma}_{2x} \mathrm{d}\bar{\sigma}_{2x} \\ &= \int_0^\infty \bar{\sigma}_{2x} \mathrm{d}P(\bar{\sigma}_{2x}) \\ &= \int_0^\infty e^{-B} \mathrm{d}\bar{\sigma}_{2x} \\ &= \int_0^\infty \exp\left[-\frac{\ell}{\ell_0}\left(\frac{\bar{\sigma}_{2x}}{\sigma_0}\right)^m I(\alpha\ell, m)\right] \mathrm{d}\bar{\sigma}_{2x} \\ &= \sigma_0 \Gamma(1 + 1/m)\left[\frac{\ell}{\ell_0} I(\alpha\ell, m)\right]^{-1/m} \end{aligned} \tag{10}$$

where $\Gamma(1 + 1/m)$ is gamma function.

For a unidirectional transverse laminate subjected to uniaxial tensile stress σ, the probability of failure $P(\sigma)$ is

$$\begin{aligned} P(\sigma) &= 1 - \exp\left[-\int_L \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m \frac{d\ell}{\ell_0}\right] \\ &= 1 - \exp\left[-\left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m \frac{L}{\ell_0}\right] \end{aligned} \tag{11}$$

where L is the length of the unidirectional laminate.

From the probability theory, the mean transverse strength of this unidirectional laminate is

$$\sigma_{mL} = \sigma_0 \Gamma(1 + 1/m)\left(\frac{L}{\ell_0}\right)^{-1/m} \tag{12}$$

Comparing Equations 10 with 12, the mean applied stress in a single segment of transverse layer can be correlated to the mean transverse strength of unidirectional laminate as follows:

$$\frac{(\bar{\sigma}_{2x})_m}{\sigma_{mL}} = \left[\frac{\alpha\ell}{\alpha L} I(\alpha\ell, m)\right]^{-1/m} \tag{13}$$

Obviously, $I(\alpha\ell, m)$ is the parameter indicating the stress nonuniformity inside the segment of transverse layer.

5.2. Prediction of cross-ply laminates

Transverse cracking problem of cross-ply laminates can be treated as the connection of multiple segments problem. Mathematically, it is the summation of non-equal length segment problem.

The probability function of composite laminates with multiple segments can be written as

$$\begin{aligned} P(\bar{\sigma}_{2x}) &= 1 - \exp\left[-\sum_i^N \int_{\ell_i} \left(\frac{\sigma_{2x} - \sigma_u}{\sigma_0}\right)^m \frac{\mathrm{d}\ell_i}{\ell_0}\right] \\ &= 1 - \exp\left(-\sum_i^N B_i\right) \end{aligned} \tag{14}$$

where

$$B_i = \frac{\ell_i}{\ell_0}\left(\frac{\bar{\sigma}_{2x}}{\sigma_0}\right)^m I(\alpha\ell_i, m) \tag{15}$$

ℓ_i is the length of the ith segment and N is the total number of segments inside the transverse layer under the applied stress $\bar{\sigma}_x$.

Similarly, The mean applied stress in the transverse layer $(\bar{\sigma}_{2x})_m$ of the cross-ply laminate is

$$\begin{aligned} (\bar{\sigma}_{2x})_m &= \int_0^\infty e^{-\sum_i B_i} \mathrm{d}\bar{\sigma}_{2x} \\ &= \sigma_0 \Gamma(1 + 1/m)\left[\sum_i \frac{\ell_i}{\ell_0} I(\alpha\ell_i, m)\right]^{-1/m} \end{aligned} \tag{16}$$

It can also be correlated to mean transverse strength of unidirectional laminate by

$$\frac{(\bar{\sigma}_{2x})_m}{\sigma_{mL}} = \left[\frac{\sum_i \alpha\ell_i I(\alpha\ell_i, m)}{\alpha L}\right]^{-1/m} \tag{17}$$

If the probability density function of segment length ℓ_i, which is the PDF of crack spacing in the composite laminate, is $f(\ell)$, Equation 17 can be rewritten as

$$\frac{(\bar{\sigma}_{2x})_m}{\sigma_{mL}} = \left[\frac{L_0 \int_0^\infty \ell I(\alpha\ell, m) f(\ell) \mathrm{d}\ell}{L \int_0^\infty \ell f(\ell) \mathrm{d}\ell}\right]^{-1/m} \tag{18}$$

where L_0 is the length of the cross-ply laminate.

Equation 18 indicates the relation between the PDF of crack spacing $f(\ell)$ and the corresponding applied stress in the laminate.

We have shown that the PDF of crack spacing is [9]

$$f(\ell) = \int_\ell^{2\ell} f(\ell, \ell_0) \cdot f_2(\ell_0) \mathrm{d}\ell_0 \tag{19}$$

where ℓ_0 is the maximum crack spacing corresponding to one specific applied stress $\bar{\sigma}_x$ as indicated in Equation 3, $f(\ell, \ell_0) = \ell_0/\ell^2$ and $f_2(\ell_0)$ is the PDF of ℓ_0.

Substituting Equations 9 and 19 into Equation 18, We can correlate the applied stress of cross-ply laminate $\bar{\sigma}_x$ to the average crack density λ in the transverse layer, where λ is the inverse of the mean crack spacing given by

$$\lambda = 1/\bar{\ell} = 1 \Big/ \int_0^\infty f(\ell)\ell \mathrm{d}\ell \tag{20}$$

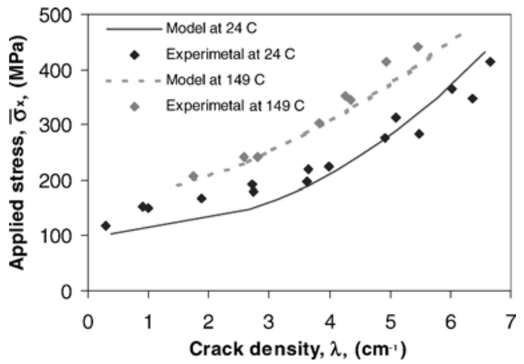

Figure 9 Comparison of applied laminate stress versus crack density between predictions and experimental results for $[0/90_4]_s$ at 24°C and 149°C.

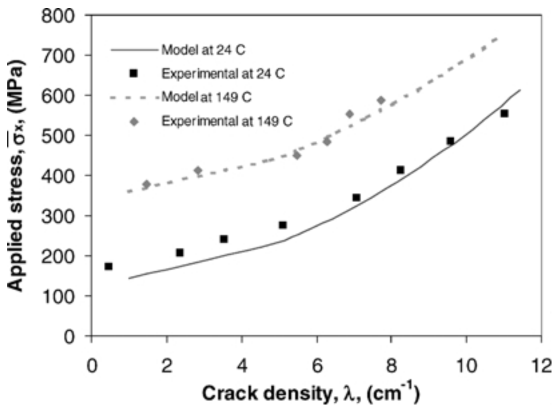

Figure 10 Comparison of applied laminate stress versus crack density between predictions and experimental results for $[0/90_2]_s$ at 24°C and 149°C.

Equations 18–20 can be evaluated by numerical software packages, such as Maple® or Mathematica®. Figs 9 and 10 show both theoretical predictions and experimental data for $[0/90_4]_s$ and $[0/90_2]_s$ at room and 149°C respectively. It shows that our model agrees reasonably well with experimental results and can be used to account for the non-deterministic matrix cracking problems.

6. Conclusions

Damage development of a high temperature polymer composite material at both room and high temperatures was studied. Since statistical scenario is always involved in the progressive matrix cracking of composite laminates due to the effects of materials defects, statistical approaches need to be introduced to accurately quantify the physics of random cracking problem. Monte Carlo simulations based on experimentally obtained parameters were employed to simulate the transverse cracking in cross-ply laminates and they agree well with the experimental results.

A novel statistical model combining Weibull theory with shear lag model was developed to predict the relation between applied stresses and crack densities. It is shown that good agreements are achieved between model predictions and experimental results.

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References

1. K. W. GARRET and J. E. BAILEY, *J. Mater. Sci.* **12** (1977) 157.
2. K. L. REIFSNIDER and J. E. MASTERS, Damage in Composite Materials, ASTM STP 775, (1982) p. 40.
3. I. M. DANIEL, J. W. LEE and G. YANIV, in Proceedings of ICCM-6 and ECCM-2, (1987) vol. 4, p. 129.
4. R. TALREJA, *J. Composite Mater.* **19** (1985) 355.
5. P. W. MANDERS, T.-W. CHOU, F. R. JONES and J. W. ROCK, *J. Mater. Sci.* **18** (1983) 2876.
6. J. W. LEE and I. M. DANIEL, *J. Composite Mater.* **24** (1990) 1225.
7. J. A. NAIRN, *ibid.* **23** (1989) 1106.
8. J. M. BERTHELOT and J. F. LE CORRE, *Composites Sci. Technol.* **60**(14) (2000) 2659.
9. Z. SUN, I. M. DANIEL and J. J. LUO, *Mater. Sci. Eng.* A **341** (2003) 49.
10. N. LAWS and G. J. DVORAK, *J. Composite Mater.* **22** (1988) 900.
11. D. R. WHEELER and H. OSAKI, American Chemical Society, ACS Symposium 440 (1990) p. 500.
12. H. D. WAGNER and A. EITAN, *Appl. Phys. Lett.* **56** (1990) 1965.
13. C. Y. HUI, S. L. PHEONIX, M. IBNABDELJALIL and R. L. SMITH, *J. Mech. Phys. Solids.* **43** (1995) 1551.
14. W. A. CURTIN, *J. Mater. Sci.* **26** (1991) 5239.
15. U. A. HANDGE, Y. LETERRIER, G. ROCHAT, I. M. SOKOLOV and A. BLUMEN, *Phys. Rev.* E **62** (2000) 7807.
16. U. A. HANDGE, *J. Mater. Sci.* **37**(22) (2002) 4803.
17. D. S. LI and M. R. WISNOM, *J. Composite Mater.* **31** (1997) 935.
18. A. R. SHAH, P. L. N. MURTHY, S. K. MITAL and R. T. BHATT, *ibid.* **34** (2000) 670.
19. Y. MICHII and J. McMANUS, *J. Reinforced Plastics Composites*, **16** (1997) 1220.
20. V. CALARD and J. LAMON, *Key Eng. Mater.* **164** (1999) 205.
21. A. S. D. WANG, *Comp. Tech. Rev.* **6** (1984) 45.
22. T. OKABE, N. TAKEDA, *et al.*, *Acta Mater*. **47** (1999) 4299.
23. A. MEZIN, J. LEPAGE, N. PACIA and D. PAULMIER, *Thin Solid Films* **172** (1989) 197.
24. N. A. WEIL and I. M. DANIEL, *J. Amer. Ceram. Soc.* **47** (1964) 268.

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